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A STUDY ON THE EFFECT OF PRANDTL NUMBER ON THE STABILITY OF THE CONDUCTION REGIME OF NATURAL CONVECTION IN AN INCLINED SLOT

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Abstract—The effect of Prandtl number on the stability of the conduction regime of natural convection in an inclined slot has been studied for the case corresponding to heating from below. For $Pr > 12{\cdot}7$ the instability occurs as waves travelling in the direction of the base flow if the angle of inclination from the vertical is small. For larger angles cells form which have their axes in the direction of the base flow. For $0{\cdot}24 < Pr < 12{\cdot}7$ the situation is similar to the above except now instead of travelling waves the instability sets in as horizontal cells for the smaller angles. The angle at which the transition between the two types of cells occurs increases from 1° to 90° as the Prandtl number decreases from 12·7 to 0·24. For $Pr < 0{\cdot}24$ only horizontal cells are possible and the stability of the flow is now mainly a function of $Pr \tan \delta$, where δ is the angle of inclination.

NOMENCLATURE

D, operator $\partial/\partial x$;
$\bar{f}$, base flow distribution in the vertical case;
g, gravity;
Gr, Grashof number;
H, height;
L, width;
Pr, Prandtl number;
Ra, Rayleigh number;
T, temperature;
T_m, mean temperature $T_m = (T(+L/2) + T(-L/2))/2$;
ΔT, temperature difference $\Delta T = T(+L/2) - T(-L/2)$;
U, characteristic thermal velocity $U = g\gamma \Delta T L^2/\nu$;
w, z-component of velocity;
x, y, z, cartesian system of coordinates.

Greek symbols

α, z-component of the wave number vector;
β, y-component of the wave number vector;
γ, coefficient of thermal expansion;
δ, angle of inclination from the vertical;
θ, $(T - T_m)/\Delta T$;
κ, thermal diffusivity;
ν, kinematic viscosity;
ρ, density;
ϕ, stream function.

Superscript

—, base flow quantity.

INTRODUCTION

THE STABILITY of the conduction regime of natural convection in an inclined slot was first discussed by Gershuni [1], who without the aid of fast computers obtained the qualitative features of the onset of instability, as well as its quantitative determination in the first approximation. More thorough studies were undertaken later by Birikh, Gershuni, Zhukhovitskii and Rudakov [2] and by Gershuni and Zhukhovitskii [3]. In [2] transverse disturbances (i.e. disturbances which are independent of the y-coordinate in the set of axes shown in Fig. 1) were considered. For these disturbances the calculation of the critical Grashof number shows that for small Prandtl numbers instability sets in as horizontal stationary cells. The stability of the flow is only slightly affected as the slot is inclined from the vertical to $\delta = 50°$, where δ is the angle of inclination from the vertical and takes on positive values when the fluid is heated from below. For larger inclination angles there is a readjustment as the marginal states of stability must follow the Rayleigh theory in the limit of $\delta = 90°$. Since this means that the Rayleigh number is constant, for the larger Prandtl numbers the critical Grashof number must drop sharply; for low Prandtl numbers the opposite is true. For fluids with $0{\cdot}22 < Pr < 0{\cdot}26$ the value of the Grashof number at the critical state remains practically unaffected. Explicit calculations of Birikh *et al.* were performed for $Pr = 0{\cdot}2$, 1·0 and 5·0.

Gershuni and Zhukhovitskii generalized the problem to include three-dimensional disturbances.

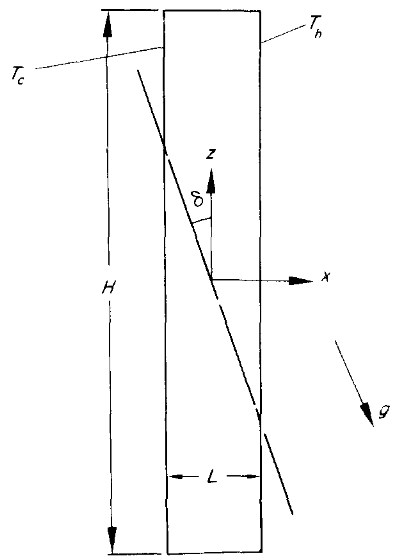

FIG. 1. Inclined slot.

These they transformed in a manner shown by Koppel [4] and by Gage and Reid [5] into two-dimensional ones, the knowledge of which allows the inverse transformation to be carried out and the behavior of the original system obtained. Their numerical results show however, that the three-dimensional case does not ever need to be considered, for the two-dimensional disturbances, whether transverse or normal to them, are always the critical ones. They also showed that the latter disturbances reduce tne analysis to that of the Rayleigh–Bénard problem with $Ra \sin \delta = 1708$ defining the onset of instability. The cells which develop in this case have their axes in the direction of the base flow. These are referred to here as longitudinal cells and the disturbances causing them as longitudinal disturbances. This kind of cellular structure is quite characteristic of the Rayleigh–Bénard convection when some type of a base flow is present. It has been observed experimentally by Chandra [6] in the case of a Couette flow which is heated from below, by Nakayama, Hwang and Cheng [7] both experimentally and numerically in the case of a heated plane Poiseuille flow, and by Liang and Acrivos [8] in the analysis of the present problem with free boundaries.

As a result of the possibility of both the transverse and longitudinal disturbances leading to instability, the actual flow pattern beyond the critical state, as noted by Gershuni and Zhukhovitskii, consists, for $Pr = 1$ and 5, of longitudinal cells which change abruptly to a horizontal cell pattern as the slot is brought from a horizontal to a vertical orientation. The angle at which the change occurs is $2{\cdot}5^\circ$ for $Pr = 5$ and 13° for $Pr = 1$. For $Pr = 0{\cdot}2$ they discovered the horizontal cells to be present for all angles.

The first experiments by De Graaf and van der Held [9] which predate the theory, show that the longitudinal cells do exist for air, at least for inclination angles close to the horizontal. Later, Kurzweg [10] working with oil of $Pr = 480$ also found agreement with the Rayleigh theory for angles of inclination $\delta > 15^\circ$. No experimental data for angles $\delta < 15^\circ$ were reported by him. Recently an extensive study dealing primarily with the convection regime was reported by Hart [11], who obtained both experimental and theoretical results for angles covering the entire range from -90° to $+90^\circ$. For the conduction regime he found that for water, $Pr = 6{\cdot}7$, the longitudinal mode sets in at $\delta > 2^\circ$. This, as well as other data presented by Hart for the conduction regime, agrees well with the results of Gershuni and Zhukhovitskii, although exact comparison is not possible as Hart considered only Prandtl numbers 0·71 and 6·7. The horizontal cells, besides being present in the inclined case persist when the slot is vertical. It was in fact in this situation in which they were first observed by Elder [12]. Later, in two very good photographs, Vest and Arpaci [13] show the qualitative features of the cells very clearly.

The experiments reported as a part of this study were performed using the apparatus constructed by Vest [14] and include the visualization of the flow pattern for air, as well as bracketing the onset of instability to a small Grashof number range. Furthermore, calculations for $Pr > 6{\cdot}7$ and for $Pr < 0{\cdot}2$ are reported as these have not been published to date except in the case of a vertical slot which was most recently studied by Korpela, Gözüm and Baxi [15]* and in the limit of infinite Prandtl number by Gill and Kirkham [16]. Finally, the proper limiting equations for low Prandtl numbers are given.

FORMULATION AND SOLUTION

Consider a slot as shown in Fig. 1, inclined from the vertical by an angle δ, with a height H and a width L to be such that the aspect ratio $H/L \gg 1$. A temperature difference ΔT is imposed between the sidewalls, considered isothermal, in such a manner that $\delta > 0$ corresponds to heating from below. The inclusion of a top and a bottom for the slot causes the contained fluid to flow parallel to the sloping sidewalls

* While this article was being reviewed a paper by Birikh *et al.* [23] appeared in English translation. The results reported in it agree well with those given in [15].

and have an antisymmetric velocity and temperature profile with respect to the $x = 0$ plane. The fluid, with a kinematic viscosity ν, thermal diffusivity κ, and a coefficient of thermal expansion γ, is treated in the Boussinesq approximation. This will require the temperature difference ΔT to be quite small in any experiment designed to verify the theory.

To write the equations governing the fluid motions in a non-dimensional form, a number of characteristic scales are introduced. The standard non-dimensionalization for natural convection includes $U = g\gamma L^2 \Delta T/\nu$ for velocity, the gap width L for length L^2/ν for time, the temperature difference ΔT for temperature and ρU^2 for pressure, where ρ is the density and g in the definition of the characteristic thermal velocity is the gravitational acceleration. The base solution (see for example Hart [10]) is then given by

$$\bar{w} \equiv \bar{f} \cos \delta = \tfrac{1}{6}(x/4 - x^3) \cos \delta \qquad \bar{\theta} = x, \quad (1)$$

where $\bar{w}$ is the z-component of the scaled velocity and $\bar{f}$ its value when the slot is vertical. The variable $\bar{\theta}$ is the nondimensional temperature measured above T_m, $\bar{\theta} \equiv (T - T_m)/\Delta T$, with T_m the mean temperature of the sidewalls.

The linearized stability equations for transverse disturbances after eliminating the pressure, introducing the stream function ϕ, and assuming periodicity in the z-direction are,

$$\frac{\partial}{\partial t}(D^2 - \alpha^2)\phi + i\alpha \, Gr \cos \delta [\bar{f}(D^2 - \alpha^2) - D^2\bar{f}]\phi$$
$$= \cos \delta D\theta + i\alpha \sin \delta \theta + (D^2 - \alpha^2)^2 \phi \quad (2)$$

$$\frac{\partial \theta}{\partial t} + i\alpha \, Gr \cos \delta \bar{f} \theta - i\alpha \, Gr \, D\bar{\theta}\phi$$
$$= \frac{1}{Pr}(D^2 - \alpha^2)\theta. \quad (3)$$

The boundary conditions to be satisfied are

$$\phi = D\phi = \theta = 0 \quad \text{at} \quad x = \pm\tfrac{1}{2}, \quad (4)$$

where $D \equiv \partial/\partial x$. The parameters present in these equations are recognized as the Grashof number $Gr = UL/\nu$, the Prandtl number $Pr = \nu/\kappa$, and the wave number α.

The longitudinal disturbances result in

$$\frac{\partial}{\delta t}(D^2 - \beta^2)\phi = i\beta \sin \delta \theta + (D^2 - \beta^2)^2 \phi \quad (5)$$

$$\frac{\partial \theta}{\partial t} = i\beta \, Gr \, D\bar{\theta}\phi + \frac{1}{Pr}(D^2 - \beta^2)\theta \quad (6)$$

$$\phi = D\phi = \theta = 0 \quad \text{at} \quad x = \pm\tfrac{1}{2} \quad (7)$$

where the wave number β gives the periodicity in the y-direction.

A look at the equations (5)–(7) reveals that the base flow velocity distribution is completely absent in this case so that if these disturbances are the critical ones the stability behavior of the flow is the same even if the velocity distribution is altered. For the heated Couette problem this is the situation as was demonstrated by Gallagher and Mercer [17] theoretically and, as previously noted, by Chandra [6] experimentally. For a slot of finite height the base velocity distribution is also altered, but due to the occurrence of a temperature gradient in the direction of the flow the equations (5)–(7) do not describe the situation properly. Nevertheless, the experiments of Hart and of Kurzweg show that the additional terms do not affect the stability substantially for $\delta > 15°$ in the range when $Pr < 480$ if the aspect ratio remains reasonably large.

If the Prandtl number is small certain terms in equations (2)–(4) become negligible. In the vertical case this leads to neglecting the temperature perturbations entirely [14]. For an inclined case the same is true except when δ approaches $\pi/2$. In that case, if $Gr \cos \delta$ and $Pr \tan \delta$ remain finite, the set (2)–(4) reduces to

$$\frac{\partial}{\partial t}(D^2 - \alpha^2)\phi + i\alpha \, Gr \cos \delta \, [\bar{f}(D^2 - \alpha^2) - D^2\bar{f}]\phi$$
$$= i\alpha \sin \delta \theta + (D^2 - \alpha^2)^2 \phi \quad (8)$$

$$\frac{1}{Pr}(D^2 - \alpha^2)\theta + i\alpha \, Gr \, D\bar{\theta}\phi = 0 \quad (9)$$

$$\phi = D\phi = \theta = 0 \quad \text{at} \quad x = \pm\tfrac{1}{2}. \quad (10)$$

Eliminating ϕ results in

$$\frac{\partial}{\partial t}(D^2 - \alpha^2)^2 \theta + i\alpha \, Gr \cos \delta [\bar{f}(D^2 - \alpha^2)$$
$$- D^2\bar{f}](D^2 - \alpha^2)\theta = +\alpha^2 \, Gr \cos \delta \, Pr \tan \delta D\bar{\theta}\theta$$
$$+ (D^2 - \alpha^2)^3 \theta \quad (11)$$

$$\theta = D^2\theta = (D^2 - \alpha^2) D\theta = 0 \quad \text{at} \quad x = \pm\tfrac{1}{2}. \quad (12)$$

The solution of equations (8)–(10) will not be carried out separately but the applicable results will be obtained from the solution of the full equations. It is merely noted here that the important parameter in this limit is seen to be the product $Pr \tan \delta$.

The Galerkin method was used to obtain a matrix eigenvalue problem which was subsequently solved numerically. The orthogonal series chosen in this method were those proposed by Vest and Arpaci [13] with the series for the stream function constructed from the C and S functions discussed by Harris and Reid [18] and a series of trigonometric functions used for the temperature. The convergence of the method as applied to the case when the slot is vertical is discussed in [15] and will not be elaborated here. As the convergence in the parameter ranges studied here is always faster than in the vertical case for the reason that the onset of instability for the inclined case with $\delta > 0$ always occurs at a lower value of the product $Gr \cos \delta$ for a given Prandtl number, the limits of accuracy reported for the vertical case form a bound for the present results.

DISCUSSION AND RESULTS

The results of our calculations are summarized in Fig. 2. The lines with slope of -1 are the result of the Rayleigh theory. In the upper portion of the figure

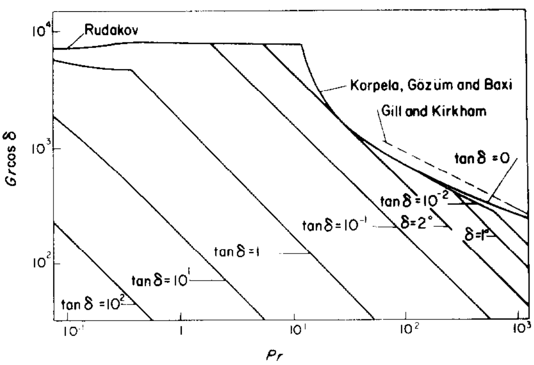

FIG. 2. The states of neutral stability as a function of Prandtl number and angle of inclination.

when $Pr < 12{\cdot}7$ the curves which are essentially horizontal give the transition to horizontal cells and for $Pr > 12{\cdot}7$ the sloping curves, $\tan \delta = 0$ and $\tan \delta = 10^{-2}$, represent cases in which instability occurs in a form of travelling waves. For the states of marginal stability in the latter region the product $GrPr^{\frac{1}{4}}$ is constant when $\delta \to 0°$, as was shown by Gill and Davey [19]. Since for the longitudinal mode the neutral states are described by the group of terms, $GrPr \sin \delta$, being constant, these two sets of curves intersect in a manner to cause the instability to always set in as transverse travelling waves for small δ followed by a transition to the longitudinal mode if the slot is inclined slightly. For $Pr = 480$, which corresponds to Kurzweg's data, the angle at which this transition occurs is about $\frac{3}{4}°$. Although there is no mention of the aspect ratio in Kurzweg's paper the criteria of either Batchelor [20] Eckert and Carlson [21] or of Gill and Davey [19] for the limit of the conduction regime, would indicate his results to correspond to the convective case. Thus the angle at which a transition between the two modes of instability takes place in his case would not correspond to our analysis. From Fig. 2 it is also noted that the marginal states of the travelling waves are affected only slightly when δ is small, the larger variations occurring at the larger Prandtl numbers. The magnitude of this deviation is sufficiently small for $Pr < 100$ and $\delta < 2°$ so that all the data in this region fall onto the $\delta = 0°$ curve. By drawing the curve of neutral stability for the longitudinal cells which is tangent to the $\delta = 0°$ curve we can deduce how the angle of transition to the longitudinal cell mode, δ_t, varies with the Prandtl number. This is shown in Fig. 3. This tangent line corresponds to $\delta = 2°$ and the

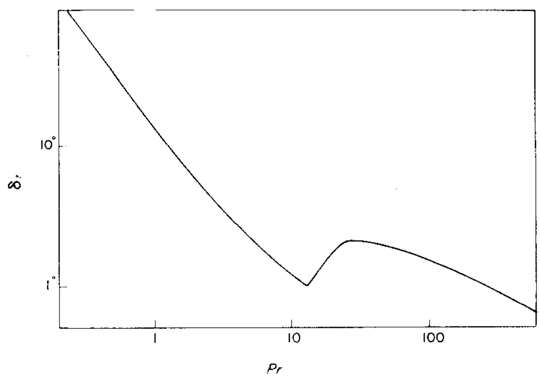

FIG. 3. The angle of transition as a function of Prandtl number.

point at tangency to $Pr = 27$, so that as the Prandtl number is increased beyond 27, the angle of transition diminishes from $\delta_t = 2°$ to 0°. On the other hand, drawing the longitudinal stability curve which passes through the intersection of the two curves of marginal states at $Pr = 12{\cdot}7$, which is the minimum Prandtl number for which travelling waves are present, it is seen that the transition angle decreases from $\delta_t = 2°$ to 1° as Pr decreases from 27 to 12·7. Below $Pr = 12{\cdot}7$ δ_t increases again.

Some results for $Pr < 12{\cdot}7$ exist in the literature. The two possible modes of instability now correspond to horizontal cells for the smaller δ and longitudinal ones beyond a certain angle. Our results for $Pr = 6{\cdot}7$ and 0·71 show the transition between the two modes to take place at $\delta_t = 1{\cdot}7°$ and 19° respectively, in agreement with Hart and for $Pr = 5$ and 1 at $\delta_t = 2{\cdot}5°$ and 13° respectively, in agreement with Gershuni and Zhukhovitskii. We mention here also the calculations of Unny [22] for Pr = 7·0 and 0·72 on the present problem, but since they do not agree with any of the other calculations now available, it appears that his results are in error.

Presentation of the results in the form as they are in Fig. 2, although concise, sacrifices some clarity. This stems from the angle of inclination being present both as a parameter and as a part of the ordinate. By increasing δ the value of $Gr \cos\delta$ defining the marginal stability is seen to decrease. The corresponding decrease in the value of $\cos\delta$ might however, be enough to offset this so that the Grashof number might actually increase. This is not the case as long as the Prandtl number is greater than about 0·24, making this value of Pr the lower limit for which the flow is destabilized monotonically with an increase in the angle of inclination. This value of Prandtl number gives also the upper limit below which only horizontal cells can be the outcome of instability so that the angle of transition has increased from $\delta_t = 1°$ to 90° as Pr decreased from 12·7 to 0·24. The effect on the stability is now for the flow to be first destabilized until a certain angle is reached and then stabilized as the slot is inclined further. The angles at which the minimum Grashof numbers occur, δ_m, are plotted in Fig. 4 as a function of the Prandtl number.

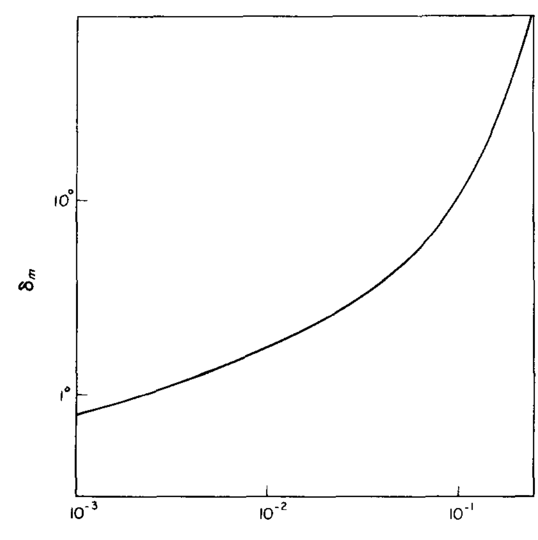

FIG. 4. The angle of maximum destabilization of the flow.

As was shown above, for small Prandtl numbers the product $Pr \tan\delta$ becomes the significant parameter. Accordingly the states of marginal stability are given with this quantity as the abscissa. From our numerical results Fig. 5 is drawn. As the Rayleigh theory is approached from below, horizontal cells result from

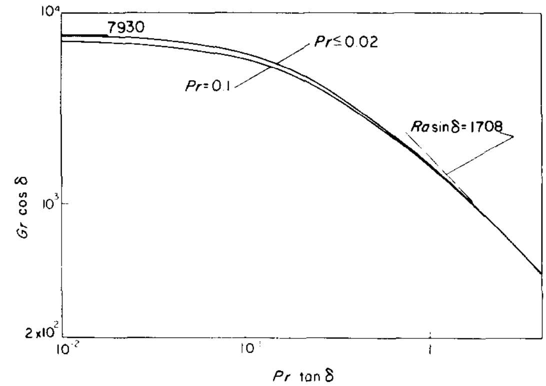

FIG. 5. The states of neutral stability for small Prandtl numbers.

instability for all states on this curve. All the data fall onto the same curve as long as $Pr \leqslant 0{\cdot}02$. The curve for $Pr = 0{\cdot}1$ is also included in this figure for comparison. The conclusion that only horizontal cells are possible for $Pr < 0{\cdot}24$ is in agreement with the results in [3] for $Pr = 0{\cdot}2$. Based on the calculation of energy integrals Hart [11] observed the mean flow to always feed energy to the transverse disturbances, this becoming a more important agent as the Prandtl number is decreased. While most of the disturbance energy still comes from the potential energy associated with the density stratification when the slot is nearly horizontal, at $Pr < 0{\cdot}24$ the transfer from the mean flow is sufficiently high to give the instability its hydrodynamic origin. This mechanism does not appear to be operative if the boundaries are free, as Liang and Acrivos [8] concluded the longitudinal modes as the only possible outcome irrespective of the Prandtl number.

An investigation of the variation of the wave number showed it not to change greatly as the slot was inclined. Any change which did occur was in the direction to decrease the wave length. In the vertical slot a typical value of α is 2·7 and as the Rayleigh–Bénard convection is approached it increases to 3·1.

EXPERIMENT

The apparatus of Vest [13] was used in the flow visualization studies. It is described by him at length, thus this is not repeated here. Introducing cigarette smoke into a layer of air enclosed within a plexiglass frame and two aluminum plates allowed visual observations of the flow patterns to be made. From recorded temperatures the critical Grashof numbers were calculated for angles corresponding to both the longitudinal and the horizontal cell modes. The results, summarized in Fig. 6, show that the theoretical

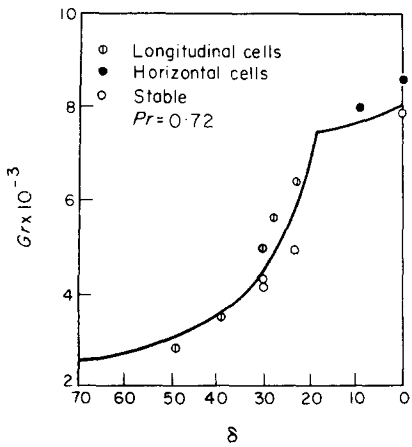

FIG. 6. Results of experiments for air ($H/L = 33$).

curve falls close to, although not entirely within the bracketed region of Grashof numbers. Two photographs of the typical cell structure are included in Figs. 7 and 8. In Fig. 7, which is similar to the one given by Vest, the Grashof number for the horizontal cells at $\delta = 0°$ is about 9000. It was found that by letting the Grashof number exceed its critical value by a reasonable amount, the smoke was entrained by the cells rapidly at a time much less than the characteristic time for diffusion, thus providing better contrast in the photograph. In this figure the air flows upward on the right hand side. The boundaries of the enclosure are identified to help distinguish the flow from its reflection on the aluminum walls.

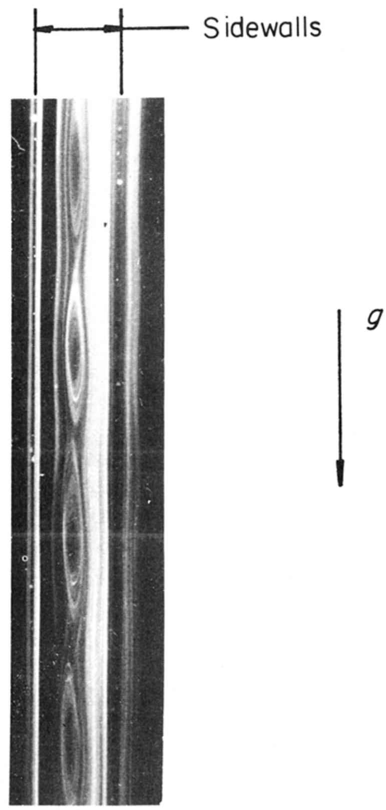

FIG. Horizontal cells of air ($H/L = 33$, $Gr = 9000$, $\delta = 0$, measured $\alpha = 2{\cdot}5$).

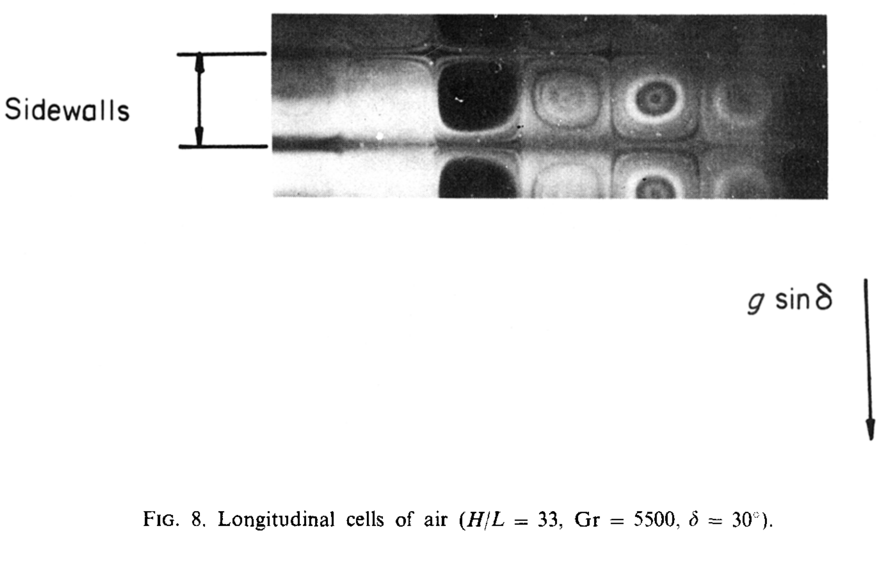

FIG. 8. Longitudinal cells of air ($H/L = 33$, Gr = 5500, $\delta = 30°$).

The longitudinal cells of Fig. 8 correspond to $\delta = 30°$ and $Gr \doteq 5500$. In this photograph the reflections of the flow from the aluminum walls are more pronounced. The wave length of this pattern is measured across two cells since the flow in any two adjacent cells are in a direction opposite to one another. From the figure it is seen that the cross section of the cells is approximately square and since the wave number for square cells is equal to π a good comparison with the theoretical value 3·12 is obtained.

SUMMARY AND CONCLUSIONS

The stability of the conduction regime of natural convection in an inclined slot heated from below can be divided into three regimes which have the following characteristics.

I. For $Pr > 12{\cdot}7$ the instability sets in as a transverse travelling wave for small angles of inclination. Longitudinal cells form as the slot is inclined beyond a certain value of δ. This angle of transition changes from 1° up to a maximum of 2° as Pr varies from 12·7 to 27. For $Pr > 27$ the angle of transition diminishes again. Inclining the slot results in the destabilization of the flow.

II. In the range $0{\cdot}24 \leqslant Pr \leqslant 12{\cdot}7$ the results established previously by others were verified.
In this range the instability sets in as horizontal cells for angles close to the vertical and leading again to a longitudinal cell pattern as the slot is inclined. The angle at which the transition occurs depends again on the Prandtl number starting from $\delta = 1°$ at $Pr = 12{\cdot}7$ and increasing to 90° as the Prandtl number is decreased to 0·24. Inclining the slot leads to destabilization of the flow in this case also.

III. For $Pr < 0{\cdot}24$ the product $Pr \tan \delta$ was shown to be the important independent parameter in the investigation of the neutral states. Only horizontal cells can result from instability in this range. Although these will occur at slightly lower values of Grashof number at first, further inclining of the slot will result in the stabilization of the flow.

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ETUDE DE L'EFFET DU NOMBRE DE PRANDTL SUR LA STABILITE DU REGIME DE CONDUCTION POUR LA CONVECTION NATURELLE DANS UNE FENTE INCLINEE

Résumé—On étudie l'effet du nombre de Prandtl sur la stabilité du régime de conduction pour la convection naturelle dans une fente inclinée dans le cas du chauffage par le bas. Pour $Pr < 12{,}7$ l'instabilité apparait comme des ondes qui se déplacent dans la direction de l'écoulement de base si l'angle d'inclinaison à partir de la verticale est petit. Pour des angles plus grands des cellules se forment qui ont leur axes dans la direction de l'écoulement de base. Pour $0{,}24 < Pr < 12{,}7$ la situation est semblable à la précédente excepté pour le parcours des ondes, l'instabilité se traduisant par des cellules horizontales pour les angles les plus petits. L'angle pour lequel se fait la transition entre les deux types de cellule subit une augmentation de 1° à 90° quand le nombre de Prandtl décroît de 12,7 à 0,24. Pour $Pr < 0{,}24$ seules les cellules horizontales sont possibles et la stabilité de l'écoulement est principalement une fonction de Pr. $\text{tg}\delta$. où δ est l'angle d'inclinaison.

STUDIE ÜBER DEN EINFLUSS DER PRANDTL-ZAHL AUF DIE STABILITÄT DES LEITUNGSREGIMES BEI NATÜRLICHER KONVEKTION IN EINEN GENEIGTEN SPALT

Zusammenfassung—Für den Fall, der Heizung von unten wurde der Einfluss der Prandtl-Zahl auf die Stabilität des Leitungsregimes bei natürlicher Konvektion in einem geneigten Spalt untersucht. Bei $Pr > 12{,}7$ tritt die Instabilität in Form von Wellen auf, die in Richtung der Grundströmung wandern, falls der Neigungswinkel gegen die Vertikale klein ist. Bei grösseren Winkeln bilden sich Zellen deren Achsen in die Richtung der Grundströmung weisen. Für $0{,}24 < \text{Pr} < 12{,}7$ ist die Situation ähnlich, nur beginnt die Instabilität bei kleineren Winkeln jetzt statt mit wandernden Wellen mit horizontalen Zellen. Der Winkel bei dem der Übergang zwischen den beiden Zellentypen stattfindet wächst von 1 auf 90° wenn die Prandtl-Zahl von 12,7 auf 0,24 abnimmt. Für $\text{Pr} < 0{,}24$ sind nur horizontale Zellen möglich und die Stabilität ist vor allem eine Funktion von $Pr \cdot tg\delta$ wobei δ der Neigungswinkel ist.

ИССЛЕДОВАНИЕ ВЛИЯНИЯ ЧИСЛА ПРАНДТЛЯ НА УСТОЙЧИВОСТЬ РЕЖИМА ТЕПЛОПРОВОДНОСТИ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В НАКЛОННОЙ ЩЕЛИ

Аннотация—Исследовалось влияние числа Прандтля на устойчивость режима теплопроводности при естественной конвекции в наклонной щели при нагреве снизу. Для $Pr > 12{,}7$ неустойчивость возникает в виде волн, которые распространяются в направлении основного течения при небольшом угле отклонения от вертикали. В случае больших углов образуются ячейки с осями, направленными в сторону основного течения. Для $0{,}24 < Pr < 12{,}7$ возникает аналогичная ситуация за исключением того, что при меньших углах наклона вместо волн неустойчивость возникает в виде горизонтальных ячеек. Угол, при котором происходит переход от одного типа ячеек к другому, увеличивается от 1° до 90°, когда число Прандтля уменьшается от 12,7 до 0,24. Для $Pr < 0{,}24$ имеют место только горизонтальные ячейки, и устойчивость течения определяется в основном величиной $Pr\, tg\delta$, где δ = угол наклона.